

**ADVANCED SUBSIDIARY GCE  
MATHEMATICS**

**4725/01**

Further Pure Mathematics 1

**FRIDAY 11 JANUARY 2008**

Morning  
Time: 1 hour 30 minutes

**Additional materials:** Answer Booklet (8 pages)  
List of Formulae (MF1)

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

- 1 The transformation  $S$  is a shear with the  $y$ -axis invariant (i.e. a shear parallel to the  $y$ -axis). It is given that the image of the point  $(1, 1)$  is the point  $(1, 0)$ .
- (i) Draw a diagram showing the image of the unit square under the transformation  $S$ . [2]
- (ii) Write down the matrix that represents  $S$ . [2]
- 2 Given that  $\sum_{r=1}^n (ar^2 + b) \equiv n(2n^2 + 3n - 2)$ , find the values of the constants  $a$  and  $b$ . [5]
- 3 The cubic equation  $2x^3 - 3x^2 + 24x + 7 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .
- (i) Use the substitution  $x = \frac{1}{u}$  to find a cubic equation in  $u$  with integer coefficients. [2]
- (ii) Hence, or otherwise, find the value of  $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$ . [2]
- 4 The complex number  $3 - 4i$  is denoted by  $z$ . Giving your answers in the form  $x + iy$ , and showing clearly how you obtain them, find
- (i)  $2z + 5z^*$ , [2]
- (ii)  $(z - i)^2$ , [3]
- (iii)  $\frac{3}{z}$ . [3]
- 5 The matrices  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  are given by  $\mathbf{A} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$  and  $\mathbf{C} = \begin{pmatrix} 2 & 4 & -1 \end{pmatrix}$ . Find
- (i)  $\mathbf{A} - 4\mathbf{B}$ , [2]
- (ii)  $\mathbf{BC}$ , [4]
- (iii)  $\mathbf{CA}$ . [2]
- 6 The loci  $C_1$  and  $C_2$  are given by
- $$|z| = |z - 4i| \quad \text{and} \quad \arg z = \frac{1}{6}\pi$$
- respectively.
- (i) Sketch, on a single Argand diagram, the loci  $C_1$  and  $C_2$ . [5]
- (ii) Hence find, in the form  $x + iy$ , the complex number represented by the point of intersection of  $C_1$  and  $C_2$ . [3]

7 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} a & 3 \\ -2 & 1 \end{pmatrix}$ .

(i) Given that  $\mathbf{A}$  is singular, find  $a$ . [2]

(ii) Given instead that  $\mathbf{A}$  is non-singular, find  $\mathbf{A}^{-1}$  and hence solve the simultaneous equations

$$\begin{aligned} ax + 3y &= 1, \\ -2x + y &= -1. \end{aligned} \quad [5]$$

8 The sequence  $u_1, u_2, u_3, \dots$  is defined by  $u_1 = 1$  and  $u_{n+1} = u_n + 2n + 1$ .

(i) Show that  $u_4 = 16$ . [2]

(ii) Hence suggest an expression for  $u_n$ . [1]

(iii) Use induction to prove that your answer to part (ii) is correct. [4]

9 (i) Show that  $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ . [2]

(ii) The quadratic equation  $x^2 - 5x + 7 = 0$  has roots  $\alpha$  and  $\beta$ . Find a quadratic equation with roots  $\alpha^3$  and  $\beta^3$ . [6]

10 (i) Show that  $\frac{2}{r} - \frac{1}{r+1} - \frac{1}{r+2} = \frac{3r+4}{r(r+1)(r+2)}$ . [2]

(ii) Hence find an expression, in terms of  $n$ , for

$$\sum_{r=1}^n \frac{3r+4}{r(r+1)(r+2)}. \quad [6]$$

(iii) Hence write down the value of  $\sum_{r=1}^{\infty} \frac{3r+4}{r(r+1)(r+2)}$ . [1]

(iv) Given that  $\sum_{r=N+1}^{\infty} \frac{3r+4}{r(r+1)(r+2)} = \frac{7}{10}$ , find the value of  $N$ . [4]

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